

PARTIAL DERIVATIVE RELATIONS FOR $f(x,y)$

$$df = \left(\frac{\partial f}{\partial x} \right)_y dx + \left(\frac{\partial f}{\partial y} \right)_x dy$$

If $f(x,y)$ also depends on z then the following relations apply:

$$1. \left(\frac{\partial f}{\partial x} \right)_z = \left(\frac{\partial f}{\partial x} \right)_y + \left(\frac{\partial f}{\partial y} \right)_x \left(\frac{\partial y}{\partial x} \right)_z$$

$$2. \left(\frac{\partial x}{\partial y} \right)_z = \frac{1}{\left(\frac{\partial y}{\partial x} \right)_z}$$

$$3. \left(\frac{\partial x}{\partial y} \right)_z = - \left(\frac{\partial x}{\partial z} \right)_y \left(\frac{\partial z}{\partial y} \right)_x$$

$$2 \text{ and } 3 \text{ combine to give Euler's Chain Relation: } \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = -1$$

$$\text{When } df(x,y) = g(x,y)dx + h(x,y)dy \text{ then } \left(\frac{\partial g}{\partial y} \right)_x = \left(\frac{\partial h}{\partial x} \right)_y$$

all the equations in no particular order

$$\text{Ch. 1 - } pV = nRT; \quad Z = \frac{V_m}{V_m^\circ}; \quad pV_m = RT \left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} \right); \quad p = \frac{nRT}{(V - nb)} - a \left(\frac{n}{V} \right)^2$$

$$\text{Ch. 2 - } \Delta U = q + w; \quad dw = -Fdz; \quad dw = -p_{ex}dV; \quad w = -p_{ex}\Delta V;$$

$$w = -nRT \ln \left(\frac{V_f}{V_i} \right)$$

$$\Delta U = q_V; \quad q = C\Delta T; \quad C = \text{calorimeter constant}; \quad C_V = \left(\frac{\partial U}{\partial T} \right)_V; \quad dU = C_V dT$$

$$H = U + pV; \quad \Delta H = q_p; \quad \Delta H = \Delta U + \Delta n_g RT; \quad C_p = \left(\frac{\partial H}{\partial T} \right)_p; \quad C_p - C_V = nR$$

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{1/c}, \quad c = \frac{C_{V,m}}{R}; \quad \Delta_{rxn} H^\ominus = \sum_{\text{products}} \nu \Delta_f H^\ominus - \sum_{\text{reactants}} \nu \Delta_f H^\ominus$$

$$\Delta_{rxn} H^\ominus(T_2) = \Delta_{rxn} H^\ominus(T_1) + \int_{T_1}^{T_2} \Delta_{rxn} C_p^\ominus dT$$

$$\mu = \left(\frac{\partial T}{\partial p} \right)_H$$

$$\text{Ch. 3 - } dS = \frac{dq_{rev}}{T}; \quad \varepsilon = \frac{|w|}{q_h}; \quad \varepsilon = 1 - \frac{T_c}{T_h}; \quad dS \geq \frac{dq}{T}; \quad \Delta_{trs}S = \frac{\Delta_{trs}H}{T_{trs}};$$

$$\Delta S = nR \ln \frac{V_f}{V_i}; \quad S(T_f) = S(T_i) + \int_{T_i}^{T_f} \frac{C_p}{T} dT; \quad \Delta S_m = C_{p,m} \ln \frac{T_f}{T_i};$$

$$\Delta S = \frac{q_{sur}}{T} = -\frac{q_{rev}}{T} = -nR \ln \frac{V_f}{V_i}; \quad S(T) = S(0) + \sum_{phases} \int_0^T \frac{C_p}{T} dT + \sum_{transitions} \frac{\Delta_{trs}H}{T_{trs}};$$

$$A = U - TS; \quad dA = dU - TdS - SdT + PdV; \quad w_{max} = \Delta A; \quad G = H - TS; \quad dG = dH - TdS - SdT + VdP; \quad w_{add, max} = \Delta G;$$

$$dU = TdS - pdV; \quad dH = TdS + VdP; \quad dA \quad \left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial p}{\partial S} \right)_V; \quad \left(\frac{\partial T}{\partial p} \right)_S = \left(\frac{\partial V}{\partial S} \right)_p;$$

$$\left(\frac{\partial p}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T; \quad \left(\frac{\partial V}{\partial T} \right)_p = - \left(\frac{\partial S}{\partial p} \right)_T; \quad \left(\frac{\partial}{\partial T} \frac{\Delta G}{T} \right)_p = - \frac{\Delta H}{T^2}; \quad \left(\frac{\partial \Delta G}{\partial p} \right)_T = \Delta V;$$

$$G(p_f) = G(p_i) + \int_{p_i}^{p_f} V dp; \quad G(p_f) = G(p_i) + V_m \Delta p;$$

$$G(p_f) = G(p_i) + nRT \ln \frac{p_f}{p_i};$$

$$\text{Ch. 4 - } \left(\frac{\partial \mu}{\partial T} \right)_p = -S_m; \quad \left(\frac{\partial \mu}{\partial p} \right)_T = V_m; \quad p = p^* e^{V_m \Delta p / RT};$$

$$\frac{dp}{dT} = \frac{\Delta_{trs}S}{\Delta_{trs}V} = \frac{\Delta_{trs}H}{T \Delta_{trs}V} \text{ (any boundary)}; \quad s-l \quad p \approx p^* + \frac{\Delta_{fus}H}{T^* \Delta_{fus}V} (T - T^*); \quad l-g$$

$$\text{Clausius-Clapeyron equation: } \frac{d \ln p}{dT} = \frac{\Delta_{vap}H}{RT^2} \quad \text{OR} \quad \ln \frac{p_f}{p_i} = - \frac{\Delta_{vap}H}{R} \left(\frac{1}{T_f} - \frac{1}{T_i} \right)$$

$$\text{OR } p = p^* e^{-\frac{\Delta_{vap}H}{R} \left(\frac{1}{T} - \frac{1}{T^*} \right)}.$$

$$\text{Ch. 5 - } V_J = \left(\frac{\partial V}{\partial n_J} \right)_{p, T, n'};$$

$$dV = \left(\frac{\partial V}{\partial n_A} \right)_{p, T, n_B} dn_A + \left(\frac{\partial V}{\partial n_B} \right)_{p, T, n_A} dn_B = V_A dn_A + V_B dn_B; \quad \mu_J = \left(\frac{\partial G}{\partial n_J} \right)_{p, T, n'};$$

$$dG = Vdp - SdT + \mu_A dn_A + \mu_B dn_B + \dots \text{ but under constant pressure and temperature -}$$

$$dG = \mu_A dn_A + \mu_B dn_B + \dots; \quad \mu_J = \left(\frac{\partial U}{\partial n_J} \right)_{S, V, n'}; \quad \mu_J = \left(\frac{\partial H}{\partial n_J} \right)_{p, S, n'};$$

$$\mu_J = \left(\frac{\partial A}{\partial n_J} \right)_{V, T, n'}; \quad \sum_J n_J d\mu_J = 0; \quad \mu = \mu^\ominus + RT \ln \frac{p}{p^\ominus}; \quad ; \text{ for the mixing of ideal}$$

gases $\Delta_{mix}G = n_A RT \ln \frac{p_A}{p} + n_B RT \ln \frac{p_B}{p} = nRT (\chi_A \ln \chi_A + \chi_B \ln \chi_B)$;

$\Delta_{mix}S = -nR (\chi_A \ln \chi_A + \chi_B \ln \chi_B)$, and $\Delta_{mix}H = 0$ Ideal-dilute solutions -

$p_B = \chi_B K_B$ (Henry's Law), Colligative Properties - $\Delta T_b = K \chi_B$ where $K = \frac{RT_b^{*2}}{\Delta_{vap}H}$,

$\Delta T_f = K' \chi_B$ where $K' = \frac{RT_f^{*2}}{\Delta_{fus}H}$, $\Pi = [B]RT$. Solvent activity, $a_A = \gamma_A \chi_A$ $\gamma_A \rightarrow 1$ as

$\chi_A \rightarrow 1$ and $\mu_A = \mu_A^* + RT \ln \chi_A + RT \ln \gamma_A$. Solute activity, for ideal dilute solution -

$\mu_B = \mu_B^* + RT \ln \frac{K_B}{p_B^*} + RT \ln \chi_B$ or $\mu_B = \mu_B^\ominus + RT \ln \chi_B$ where $\mu_B^\ominus = \mu_B^* + RT \ln \frac{K_B}{p_B^*}$.

For real solutions, $\mu_B = \mu_B^\ominus + RT \ln a_B$ where $a_B = \gamma_B \chi_B$ and $\gamma_B \rightarrow 1$ as $\chi_B \rightarrow 0$. Can also define activities in terms of molalities so that $a_B = \gamma_B \frac{b_B}{b^\ominus}$.

Ch. 6 – The Phase Rule: $F = C - P + 2$.

Ch. 7 - $\Delta G_{rxn} = \left(\frac{\partial G}{\partial \xi} \right)_{p,T} = \mu_B - \mu_A$, $\Delta G_{rxn}^\ominus = \mu_B^\ominus - \mu_A^\ominus = \sum_J \nu_J \Delta G_f^\ominus (J)$. At any point

in a reaction sequence: $\Delta G_{rxn} = \Delta G_{rxn}^\ominus + RT \ln Q$, where $Q = \prod_J a_J^{\nu_J}$. BUT at equilibrium

$\Delta G_{rxn} = 0 = \Delta G_{rxn}^\ominus + RT \ln K$, where $K = \left(\prod_J a_J^{\nu_J} \right)_{equilibrium}$. Responses to stresses:

$\left(\frac{\partial K}{\partial p} \right)_T = 0$. So that when α is the extent of dissociation partial pressures respond such that

$\alpha = \left(\frac{1}{1 + 4p/K} \right)^{1/2}$. The van't Hoff eqn.: $\left(\frac{d \ln K}{dT} \right) = \frac{\Delta H_{rxn}^\ominus}{RT^2}$ or $\left(\frac{d \ln K}{d \left(\frac{1}{T} \right)} \right) = -\frac{\Delta H_{rxn}^\ominus}{R}$ so

that $\ln K_2 - \ln K_1 = -\frac{\Delta H_{rxn}^\ominus}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$.

$\Delta_r G = -nFE$; $E = E^\circ - \frac{RT}{nF} \ln Q$; $\Delta_r G^\circ = -nFE^\circ$; $E^\circ = \frac{RT}{nF} \ln K$

Ch. 22 - $\text{Rate} = \frac{d\xi}{dt} = \frac{1}{\nu_J} \frac{dn_J}{dt} = \frac{1}{\nu_J} \frac{d[J]}{dt}$. First order reactions: $\frac{d[A]}{dt} = -k[A]$;

$\ln\left(\frac{[A]}{[A]_0}\right) = -kt$ or $[A] = [A]_0 e^{-kt}$. $t_{1/2} = \frac{\ln 2}{k}$ and $\tau = \frac{1}{k}$. Second order reactions

$(2A \rightarrow P)$: $\frac{d[A]}{dt} = -k[A]^2$;

$\frac{1}{[A]} - \frac{1}{[A]_0} = kt$ or $[A] = \frac{[A]_0}{1 + kt[A]_0}$ with $t_{1/2} = \frac{1}{k[A]_0}$. Arrhenius Equation -

$\ln k = \ln A - \frac{E_a}{RT}$ or $k = Ae^{-\frac{E_a}{RT}}$ For $A \xrightarrow{k_a} I \xrightarrow{k_b} P$, the steady state approximation

assumes $\frac{d[I]}{dt} = 0$